Hidden Symmetry, Excitonic Transitions and Two-Dimensional Kane's Exciton in the Quantum Well

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Abstract

In this article it is shown that, Sommerfeld's coefficients for excitonic transitions in quantum wells are determined only with the principle quantum number within the framework of two-dimensional Coulomb potential. This is a consequence of hidden symmetry of two-dimensional Coulomb problem, conditioned by the existence of two-dimensional analog of the Runge-Lentz vector. For the narrow gap semiconductor quantum well with the non-parabolic dispersion law of electron and hole in the two-band Kane model it is shown that two-dimensional excitonic states are described in the frames of an analog of Klein-Gordon equation with the two-dimensional Coulomb potential. The non-stability of the ground state of the two-dimensional Kane's exciton is show.

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1. Introduction

Due to the existence of the size quantization effects in the semiconductor nanostructures it became possible to realize such a low dimensional systems, which initially had only model character. One of such is the two-dimensional Coulomb system, which is obtained during the impurity and excitonic states formation in quantum wells and superlattices [1-8]. Mathematically two-dimensional Coulomb potential is defined by the following expression [9]:

$$V(x,y) = -\frac{Ze^2}{\sqrt{x^2+y^2}}, \tag{1}$$

where $Z-$ is the charge number. Corresponding Schrödinger equation in the Cartesian coordinates is the following

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\Psi - \frac{Ze^2}{\sqrt{x^2+y^2}}\Psi = E\Psi. \tag{2}$$

It should be mentioned that two-dimensional hydrogen atom, being a system with the hidden symmetry, can be investigated simultaneously in several coordinate systems as: polar [10], parabolic [11] and elliptical [12]. In the absence of fields, it is more appropriate to use polar coordinate system.

As it was mentioned, one of the causes for the formation of two-dimensional Coulomb systems in semiconductor quantum wells is the formation of excitons in them during the interband optical absorption [13]. Since the system under is two-dimensional the exciton in it is also two-dimensional (provided strong size quantization [14]) and the energy of electron-hole interaction can be described in the frames of two-dimensional Coulomb potential (1) and charge number $Z = 1$. Two-dimensional excitonic systems can be realized also in quantum dots in the presence of strong vertically quantization or magnetic field (see f. e. [15-17]). Thus, two-dimensional excitonic states will also have the hidden symmetry, natural for the two-dimensional Coulomb problem. It is clear that this fact should be reflected on the character of the excitonic transitions in quantum well.

On the other hand it should be mentioned that not all semiconductor quantum wells have the parabolic dispersion law of the charge carriers. There are compounds in which, due to the existence of strong interaction of valence band with the band of conductivity, non parabolic dispersion law for the electrons and holes can exist [18]. For that case, theoretical description of the two-dimensional excitons should be done with the consideration of the kinetic operator, which deviates from standard quadratic. It is natural to expect that the above mentioned case can bring to the disappearance of the occasional degeneration of the two-dimensional Coulomb problem discussed.

In this article we study the influence of the hidden symmetry on the Sommerfeld coefficient for excitonic transitions in semiconductor quantum wells. We also investigate two-dimensional excitonic states in narrowband semiconductor quantum wells with the Kane's dispersion low for charge carriers.

2. Two-dimensional excitonic states

As was mentioned above we are studying the two-dimensional electron-hole system with the interaction potential (1). Corresponding Schrödinger two-particle equation will have the following form.

$$-\left\{\frac{\hbar^2}{2m_e}\Delta_e + \frac{\hbar^2}{2m_h}\Delta_h + \frac{e^2}{|\vec{\rho}_e - \vec{\rho}_h|}\right\}\Psi = E\Psi, \tag{3}$$

where $m_{e(h)}-$is the effective mass of the electron(hole) and $\frac{\hbar^2}{2m_{e(h)}}\Delta_{e(h)}-$ is the kinetic energy of the electron(hole) operator. Introducing reduced mass

$$\mu = \frac{m_e m_h}{m_e + m_h},$$

Relative coordinate

$$\vec{\rho} = \vec{\rho}_e - \vec{\rho}_h,$$

and applying standard procedure for the transition from two- to one-particle problem we will obtain two-dimensional Schrödinger equation

$$-\left\{\frac{\hbar^2}{2\mu}\Delta + \frac{e^2}{\rho}\right\}\Psi = E\Psi. \tag{4}$$

In polar coordinates equation (4) will get to the following form [9]

$$-\frac{\hbar^2}{2\mu}\left\{\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2}{\partial\varphi^2}\right\}\Psi_{n,m}(\rho,\varphi) - \frac{e^2}{\rho}\Psi_{n,m}(\rho,\varphi) = E\Psi_{n,m}(\rho,\varphi). \tag{5}$$

The well-known solution for this equation is following

$$\Psi_{n,m}(\rho,\varphi) = \frac{e^{im\varphi}}{\sqrt{2\pi}}R_{n,m}(\rho), \tag{6}$$

where

$$R_{n,m}(\rho) = \left[\frac{2(n-|m|)!}{a_{ex}^2\left(n+\frac{1}{2}\right)^3\{(n+|m|)!\}^3}\right]^{\frac{1}{2}}\left(\frac{\rho}{a_{ex}\lambda}\right)^{|m|}e^{-\left(\frac{\rho}{2a_{ex}\lambda}\right)}L_{n+|m|}^{2|m|}\left(\frac{\rho}{a_{ex}\lambda}\right), \tag{7}$$

and where $a_{ex} = \frac{\hbar^2}{\mu e^2}-$is the exciton radius, $\lambda = \sqrt{-\frac{Ry^*}{4E}}$, Ry^*- is the effective Rydberg's energy, $m-$is the magnetic quantum number, $n = n_\rho + |m| -$is the principle quantum number ($m = 0, \pm1, \pm2, ..., \pm n$), $n_\rho-$is the radial quantum number, $L_n^\alpha(x) -$is the Laguerre's polynomial. Corresponding energetic spectrum will be

$$E_n^{ex} = -\frac{\mu e^4}{2\hbar^2\left(n+\frac{1}{2}\right)^2}. \tag{8}$$

As it can be seen from (8), two-dimensional excitonic states are degenerated with the degeneracy order

$$g(n) = 2n + 1. \tag{9}$$

Instead of expected $O(2)$ rotational symmetry (with respect to z axis) given by the matrix

$$R(\varphi) = \begin{pmatrix} \cos\varphi & -\sin\varphi & 0 \\ \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{10}$$

the considered system must possess more large symmetry. In the paper [19] it is shown, that this degeneration is connected with the existence of the two-dimensional analog of the Runge-Lentz vector in the two-dimensional Coulomb problem. Components of that vector are

$$\begin{aligned}\hat{\gamma}_1 &= \frac{1}{\sqrt{-2E}}\left(\frac{x}{\rho} - \hat{P}_y\hat{L}_z + i\frac{\hat{P}_x}{2}\right),\\ \hat{\gamma}_2 &= \frac{1}{\sqrt{-2E}}\left(\frac{y}{\rho} + \hat{L}_z\hat{P}_x - i\frac{\hat{P}_y}{2}\right).\end{aligned} \tag{11}$$

These operators along with plane rotation generator $\hat{\gamma}_3 = \hat{L}_z = -i\hbar\frac{\partial}{\partial\varphi}$ Abelian group $O(2)$ define hidden group symmetry $SO(3)$ commutating with the Hamiltonian of the two-dimensional Coulomb problem and providing corresponding degeneracy order. At the same time generators of this group satisfy the commutation relation

$$[\hat{\gamma}_i\hat{\gamma}_j] = i\varepsilon_{ijk}\hat{\gamma}_k, \tag{12}$$

where ε_{ijk} is the absolute anti-symmetric tensor.

3. Sommerfeld coefficients and excitonic transitions in quantum well

During the investigation of the optical characteristics of the semiconductors it was revealed that under the certain conditions of the light absorption in semiconductors may be obtained such a conditions in which electron transiting from valence band to the conductivity band can create a bound state with the hole in the valence band, without reaching the conductivity band [14]. In other words excitonic state occurs and corresponding transitions are called excitonic. In the theory of semiconductor optical properties it is shown that the intensity of the excitonic transitions is characterized with the Sommerfeld coefficients [14], which are equal to the values of the squared modulus of excitonic wave function in $\rho = 0$

$$Z_1(n) = |\Psi_{n,m}(0)|^2, \tag{13}$$

if $\Psi_{n,m}(0) \neq 0$, and

$$Z_2(n) = |grad\Psi_{n,m}(0)|^2, \tag{14}$$

if $\Psi_{n,m}(0) = 0$. For the first case transitions are called allowed, and for the second case – forbidden.

Let's define the Sommerfeld coefficients for the excitonic transitions in the quantum well, when exciton is described in the frames of above mentioned two-dimensional Coloumbic problem. We should use the wave function (7). From that it is obvious that allowed transitions take place only for the states for which$m = 0$. Thus the corresponding Sommerfeld's coefficient will have the following form [14]

$$Z_1(n) = |\Psi_{n,0}(0)|^2 = \frac{1}{\pi a_{ex}^2\left(n+\frac{1}{2}\right)^3}. \tag{15}$$

For the forbidden transitions case, first, we should write Laguerre's polynomial in the waive function in the (7) explicit form

$$L_{n+|m|}^{2|m|}(x) = \sum_{k=0}^{n-|m|} \frac{(-1)^{k+2|m|}\left\{(n+|m|)!\right\}^2 x^k}{(n-|m|-k)!\,(2|m|+k)!k!}. \tag{16}$$

Direct calculations show that $grad\Psi_{n,m}(0)$ is nonzero when $m=0$ and $m=\pm 1$. So for the Sommerfeld's coefficient we can take

$$Z_2(n) = \sum_{i=-1}^{1} |grad\Psi_{n,i}(0)|^2. \tag{17}$$

Calculating the corresponding values for the gradients we can see that

$$|grad\Psi_{n,0}(0)|^2 = \frac{4\left(n+\frac{1}{2}\right)^2}{\pi a_{ex}^4\left(n+\frac{1}{2}\right)^5}, \tag{18}$$

$$|grad\Psi_{n,\pm 1}(0)|^2 = \frac{n(n+1)}{\pi a_{ex}^4\left(n+\frac{1}{2}\right)^5}. \tag{19}$$

Taking into account (17-19) for the Sommerfeld's coefficient in the case of forbidden transitions we get:

$$Z_2(n) = \frac{3\left(n+\frac{1}{2}\right)^2 - \frac{1}{4}}{2\pi a_{ex}^4\left(n+\frac{1}{2}\right)^5}. \tag{20}$$

As it follows the intensity of the forbidden excitonic transitions in the two-dimensional case depends only on the principle quantum number n. For its fixed values, due to the existence of hidden symmetry of the discussed problem, for the states with different m and n_ρ, but with the same $n = n_\rho + |m|$ we get the same intensity for the excitonic transitions $Z_2(n)$. In Table 1 Sommerfeld's coefficients for different forbidden transitions are brought.

$n=1$	$n=2$	$n=3$
$n_\rho=1,\ m=0$	$n_\rho=2,\ m=0$	$n_\rho=3,\ m=0$
$n_\rho=0,\ m=1$	$n_\rho=1,\ m=1$	$n_\rho=2,\ m=1$
$n_\rho=0,\ m=-1$	$n_\rho=1,\ m=-1$	$n_\rho=2,\ m=-1$
$Z_2(1)=\frac{1}{\pi a_{ex}^4}\frac{104}{243}$	$Z_2(2)=\frac{1}{\pi a_{ex}^4}\frac{296}{3125}$	$Z_2(3)=\frac{1}{\pi a_{ex}^4}\frac{584}{16807}$

Table 1: Sommerfeld's coefficients for the forbidden two-dimensional excitonic transitions for different values of main quantum number n.

To sum up, we can conclude that for the strong quantization in quantum well, when we can use two-dimensional exciton model, due to the hidden symmetry of the two-dimensional Coulomb problem, Zommerfield's coefficients of the exciton transitions are expressed only with

principle quantum number. This makes possible for the case of forbidden transitions to realize the same intensity of the two-dimensional excitonic transitions for different states. As soon as we assume the quasi two-dimensionality of the exciton and insert the z coordinate into Hamiltonian the hidden symmetry of the problem disappears and Sommerfeld's coefficients now depend also on m and n_ρ.

4. Two-dimensional Klein-Gordon equation and Kane's exciton in quantum well

Along with the consideration of quasi two-dimensionality, also consideration of non-parabolicity of dispersion law for the electron and hole in the narrow band semiconductor quantum wells in two-dimensional Coulomb problem can bring to the disappearance of the hidden symmetry. What is important, that electron-whole interaction potential can be described within the frames of two-dimensional potential (1). In Kane's works it was shown that consideration of interband interactions in semiconductor brings to the deviation of the electron-whole dispersion law from the quadratic form [18]. In two-band approximation, when the effective masses of electron and whole are equal, Kane's dispersion law becomes analogous to the relativistic one, however, naturally, there is nothing relativistic, simple mathematical coincidence. If we introduce the band interaction parameter $s(s \approx 10^8$ sm/s), which is defined through interband dipole matrix element, the corresponding dispersion law will take the following form [18]:

$$E = \sqrt{p^2 s^2 + \mu^2 s^4}. \tag{21}$$

Thus, within the frames of two-band Kane's approximation for defining the excitonic states one does have to solve corresponding steady-state Klein-Gordon equation with the Coulomb potential. It is clear, that for the two-dimensional excitonic states in narrowband semiconductor quantum well problem reduces to the investigation of the two-dimensional Klein-Gordon equation with the Coulombic interaction term (1).

In the polar coordinates Klein-Gordon equation for the Coulomb field takes the following form [20]

$$\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2}{\partial\varphi^2}\right]\Psi_{n_\rho,m} + \left[\frac{2ZEe^2}{\hbar^2c^2\rho} + \frac{Z^2e^4}{\hbar^2c^2\rho^2} - \frac{1}{\hbar^2c^2}\left(\mu^2c^4 - E^2\right)\right]\Psi_{n_\rho,m} = 0, \tag{22}$$

where μ is the electron mass.

Representing electron's wave function as a compound of radial $R(\rho)$ and angular $\Phi(\varphi)$ functions $\Psi_{n_\rho,m}(\rho,\varphi) = R_{n_\rho,m}(\rho)\,\Phi_m(\varphi)$ and using the variable separation method, from (1) we obtain two equations which define $R(\rho)$ and $\Phi(\varphi)$:

$$\rho^2\frac{d^2R_{n_\rho,m}}{d\rho^2} + \rho\frac{dR_{n_\rho,m}}{d\rho} + \left[\frac{2Ze^2E}{\hbar^2c^2}\rho + \frac{Z^2e^4}{\hbar^2c^2} - \frac{(\mu^2c^4 - E^2)}{\hbar^2c^2}\rho^2 - m^2\right]R_{n_\rho,m} = 0, \tag{23}$$

and

$$\frac{d^2\Phi_m}{d\varphi^2} + m^2\Phi_m = 0, \tag{24}$$

where $m = 0; \pm 1; \pm 2;$ ... is the magnetic quantum number, $n_\rho-$is the radial quantum number.

From the (24) for the normalized angular functions $\Phi_m(\varphi)$ we get

$$\Phi_m(\varphi) = \frac{1}{\sqrt{2\pi}} e^{im\varphi}. \tag{25}$$

For the radial equation (23) first, let's introduce following notations:

$$\varepsilon = \frac{1}{\hbar c}\left(\mu^2 c^4 - E^2\right)^{1/2}, \quad \lambda = Z\alpha E\left(\mu^2 c^4 - E^2\right), \quad \alpha = \frac{e^2}{\hbar c}, \tag{26}$$

and dimensionless variable $r = 2\varepsilon\rho$. Then for the $R(r)$we come to the following equation

$$R''_{n_\rho,m} + \frac{R'_{n_\rho,m}}{r} + \left(\frac{\lambda}{r} + \frac{Z^2\alpha^2 - m^2}{r^2} - \frac{1}{4}\right) R_{n_\rho,m} = 0. \tag{27}$$

Below we find the solutions of equations (27) in two limiting cases: $r \to 0$ and $r \to \infty$. Then the equation for $R(r)$ is reduced to

$$R' + \frac{1}{r}R' + \frac{Z^2\alpha^2 - m^2}{r^2} R = 0. \tag{28}$$

We look for $R(r)$ in the form r^S. Substituting this into (28) we obtain the following quadratic equation for S

$$S(S-1) + S + Z^2\alpha^2 - m^2 = 0. \tag{29}$$

The two roots are:

$$S_1 = \sqrt{m^2 - Z^2\alpha^2}, \quad S_2 = -\sqrt{m^2 - Z^2\alpha^2}.$$

Only the first root guaranties the convergence of $R(r)$ at $r \to 0$. Finally, we have:

$$R(r) = R_0(r) \sim r^{\sqrt{m^2 - Z^2\alpha^2}}. \tag{30}$$

$r \to \infty$.In this case the equation (28) take the form:

$$R' - \frac{1}{4}R = 0. \tag{31}$$

The solution obeying the standard conditions is:

$$R(r) = R_\infty(r) \sim e^{-r/2}. \tag{32}$$

In general case we apply for $R(r)$ the ansatz $R_\infty \cdot R_0 \cdot u(r)$, where $u(r)$ is unknown function. So,

$$R(r) = r^S \exp(-r/2)\, u(r). \tag{33}$$

Substituting (33) in (27) we get for $u(r)$ Kummer equation [21]:

$$ru' + (2S + 1 + r)\, u' + \left(\lambda - S - \frac{1}{2}\right) u = 0. \tag{34}$$

We will find the solution of (34) in the form of power series:

$$u(r) = \sum_{k=0}^{\infty} a_k r^k. \tag{35}$$

Substituting (35) in (34) and gathering together the terms with the same order of r, we arrive at:

$$\sum_k \left[(k+1)\,(k+2S+1)\, a_{k+1} + \left(\lambda - S - \frac{1}{2} - k\right) a_k\right] r^k = 0. \tag{36}$$

From (38) we obtain the following recurrent relation for the coefficients of (35):

$$a_{k+1} = \frac{\left(k + S + \frac{1}{2} - \lambda\right)}{(k+1)\,(k+2S+1)} a_k. \tag{37}$$

It is clear from this relation that at $r \to \infty$ the series (35) diverges as e^r, and for its convergence the relation $\lambda = k + S + \frac{1}{2}$ must be imposed. Using (26), it can be presented as:

$$\frac{Z\alpha E}{\sqrt{\mu^2 c^4 - E^2}} = k + S + \frac{1}{2}. \tag{38}$$

Solving the equation (38) with respect to E, we obtain the energy spectra of atom:

$$E_{k,m} = \frac{\left(k + S + \frac{1}{2}\right)\mu c^2}{\left[Z^2\alpha^2 + \left(k + S + \frac{1}{2}\right)^2\right]^{1/2}} = \mu c^2 \left[1 - \frac{Z^2\alpha^2}{Z^2\alpha^2 + \left(k + \frac{1}{2} + \sqrt{m^2 - Z^2\alpha^2}\right)^2}\right]^{1/2}. \tag{39}$$

Finally, the solution of the (27), which satisfies standard conditions, can be expressed through the confluent hyper geometrical function ${}_1F_1\,(\alpha, \beta; x)$. Thus for the radial waive function we can take

$$R_{n_\rho,m}\,(r) = C_{n_\rho,m} r^S \exp(\,-r/2)\;\; {}_1F_1\left(S + \frac{1}{2} - \lambda;\; 2S+1;\; r\right), \tag{40}$$

where $C_{n_\rho,m}-$ is the normalization constant.

As it follows from (39), the value of energy for $m = 0$ becomes complex. Otherwise an instability, similar to one in three-dimensional Klein-Gordon hydrogen atom at $Z\alpha > \frac{1}{2}$ [22-23], appears in the system.

The relation (39) could be obtained comparing of radial equations for relativistic and non-relativistic problems and using the exact form of energy spectra for non-realtivistic case. The equations are:

$$\begin{aligned} &-\frac{\hbar^2}{2\mu}\frac{1}{\rho}\frac{d}{d\rho}\left(\rho\frac{d}{d\rho}\right)R+\frac{\hbar^2 m^2}{2\mu\rho^2}R-\frac{Ze^2}{\rho}R=ER, \\ &-\frac{\hbar^2}{2\mu}\frac{1}{\rho}\frac{d}{d\rho}\left(\rho\frac{d}{d\rho}\right)R+\frac{\hbar^2\left(m^2-Z^2\alpha^2\right)}{2\mu\rho^2}R-\frac{Ze^2E}{\rho\mu c^2}R=\frac{\left(E^2-\mu^2c^4\right)}{2\mu c^2}R. \end{aligned} \tag{41}$$

It is easy to see that the second equation is reduced to the first one after the following transformations:

$$m^2\to m^2-Z^2\alpha^2,\quad Z\to\frac{ZE}{\mu c^2},\quad E\to\frac{E^2-\mu^2c^4}{2\mu c^2}.$$

Substituting these notations into the expression for the energy of non-relativistic two-dimensional hydrogen atom, we arrive at (39).

As it was mentioned, Kane's dispersion law of the charge carriers in the two-band approximation has the form analogous to the relativistic one. That is why we can reduce the two-particle Hamiltonian function of the exciton problem to the effective one-particle one, and apply the results obtained above to the Kane's exciton.

For the Kane's exciton case the role of the light speed plays the parameter$s$, mass of the free electronμ replaces with the effective mass μ_e of the electron in the crystal. From that for the effective constant of the fine structure α^* we can take

$$\alpha^*=\frac{e^2}{\hbar s}. \tag{42}$$

In the two-band interaction effective masses of the electron and hole are equal,$\mu_e=\mu_h$, which makes it possible to bring the two-particle Hamilton function to the one-particle. Indeed, passing to the new coordinate system, in which $\mu_e\vec{\rho}_e+\mu_h\vec{\rho}_h=0$, and introducing $\vec{\rho}=\vec{\rho}_e-\vec{\rho}_h$, as well as considering that $\vec{\rho}_e=\frac{\mu_h}{\mu_e+\mu_h}\vec{\rho}$ and $\vec{\rho}_h=-\frac{\mu_e}{\mu_e+\mu_h}\vec{\rho}$, for the electron and hole impulse operators we will get

$$\begin{aligned} \hat{\vec{P}}_e&=\frac{\mu_e+\mu_h}{\mu_h^*}\hat{\vec{P}} \\ \hat{\vec{P}}_h&=-\frac{\mu_e+\mu_h}{\mu_e}\hat{\vec{P}}, \end{aligned} \tag{43}$$

where $\hat{\vec{P}}=-i\hbar\frac{\partial}{\partial\vec{\rho}}$. Switching to the new coordinate $\vec{\rho}'=\frac{\vec{\rho}}{4}$ and introducing effective mass and charge, $\mu'\equiv 2\mu$, $e'\equiv 2e$ we come to the following equation

$$\left(\hat{P}'^{\,2}s^2+\mu'^{\,2}s^4\right)\Psi_{n_\rho,m}=\left(E^{ex}_{n_\rho,m}+\frac{e'^{\,2}}{\rho'}\right)^2\Psi_{n_\rho,m} \tag{44}$$

which fully coincides with the Klein-Gordon equation for the two-dimensional hydrogen-like atom, where instead of μ we have μ', $e-e'$, $c-s$, and $\hat{\vec{P}}' = 2\hat{\vec{P}}$. This allows us to apply to the Kane's exciton main results, obtained during the solution of the two-dimensional relativistic hydrogen atom which are:

Exciton's energetic spectrum has the form

$$E^{ex}_{n_\rho,m} = \left[1 - \frac{4\alpha^{*2}}{4\alpha^{*2} + \left(n_\rho + \frac{1}{2} + \sqrt{m^2 - 4\alpha^{*2}}\right)^2}\right] \mu' s^2, \tag{45}$$

The states of the system with $m = 0$ are unstable.

Degeneration of energy levels, which takes place in the case of two-dimensional exciton with the standard dispersion law, disappears.

Finally, after the standard procedure of the normalizations of the radial part of the exciton's wavefunction

$$\int_0^\infty R^2_{n_\rho,m}(\rho)\,\rho d\rho = 1, \tag{46}$$

and using the equation

$$\begin{aligned} I = \int_0^\infty e^{-\chi x} x^{\nu-1} F^2(-n;\gamma;\chi x)\,dx = \frac{\Gamma(\nu)\,n!}{\chi^\nu \gamma(\gamma+1)\cdots(\gamma+n-1)} \\ \times \left\{1 + \sum_{p=0}^{n-1} \frac{n(n-1)\cdots(n-p)(\gamma-\nu-p-1)\cdots(\gamma-\nu+p)}{[(p+1)!]^2\,\gamma(\gamma+1)\cdots(\gamma+p)}\right\}, \end{aligned} \tag{47}$$

for $\Psi_{n_\rho,m}(\rho,\varphi) = R_{n_\rho,m}(\rho)\,\Phi_m(\varphi)$ we obtain:

$$\begin{aligned} \Psi_{n_\rho,m}(\rho,\varphi) = \frac{1}{a_{ex}} \sqrt{\frac{2(2S+1)\ldots(2S+n_\rho)}{\pi\,\Gamma(2S+2)\,n_\rho!\left(1+\frac{2n_\rho}{2S+1}\right)\left(Z^2\alpha^2 + \left(n_\rho + S + \frac{1}{2}\right)^2\right)}} \\ \times e^{im\varphi} e^{-\varepsilon\rho} (2\varepsilon\rho)^S F(-n_\rho;\,2S+1;\,2\varepsilon\rho). \end{aligned} \tag{48}$$

4. Conclusion

So, the presence of hidden symmetry in two-dimensional Coulomb system affects the character of forbidden exciton transitions in narrowband quantum wells in the case when the system can be considered as an exact two-dimensional one. The quasi-two-dimensionality or non-parabolisity of the dispersion low takes away this hidden symmetry, and the explicit dependence on magnetic quantum number m appears. For two-band Klein model applied to exciton levels (in this case the dispersion low of charge carriers is similar to the relativistic one) apart from

the removal of level degeneracy with respect to m, instabilities for all energy levels since $m = 0$ appear in this problem.

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